

GLUEBALLS, HYBRID AND EXOTIC MESONS AND STRING BREAKING

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We review lattice QCD results for glueballs (including a discussion of mixing with scalar mesons), hybrid mesons and exotic mesons (such as $B_s B_s$ molecules). We also discuss string breaking as a mixing between colour flux states and $B\bar{B}$ states.

1 Introduction

The most systematic approach to non-perturbative QCD is via lattice techniques. Lattice QCD needs as input the quark masses and an overall scale (conventionally given by Λ_{QCD}). Then any Green function can be evaluated by taking an average of suitable combinations of the lattice fields in the vacuum samples. This allows masses to be studied easily and matrix elements (particularly those of weak or electromagnetic currents) can be extracted straightforwardly. Unlike experiment, lattice QCD can vary the quark masses and can also explore different boundary conditions and sources. This allows a wide range of studies which can be used to diagnose the health of phenomenological models as well as casting light on experimental data.

One limitation of the lattice approach to QCD is in exploring hadronic decays because the lattice, using Euclidean time, has no concept of asymptotic states. One feasible strategy is to evaluate the mixing between states of the same energy - so giving some information on on-shell hadronic decay amplitudes.

For comparison with models and for ease of computation, the special case of infinitely heavy sea quarks (namely neglect of quark effects in the vacuum: the quenched approximation) is often used. We shall also present results from including sea quark effects - usually two flavours of degenerate sea quark of mass equivalent to strange quarks or heavier.

The quark model gives a good overall description of hadronic spectra. Here I will discuss lattice results for states which go beyond the quark model: glueballs, exotic mesons and hybrid mesons. I also discuss the related phenomenon of string breaking.

2 Glueballs

Glueballs are defined to be hadronic states made primarily from gluons. The full non-perturbative gluonic interaction is included in quenched QCD. In the quenched approximation, there is no mixing between such glueballs and quark - antiquark mesons. A study of the glueball spectrum in quenched QCD is thus of great value. This will allow experimental searches to be guided as well as providing calibration for models of glueballs. A non-zero glueball mass in quenched QCD is the “mass-gap” of QCD. To prove this rigourously is one of the major challenges of our times. Here we will explore the situation using computational techniques.

In principle, lattice QCD can study the meson spectrum as the sea quark mass is decreased towards experimental values. This will allow the unambiguous glueball states in the quenched approximation to be tracked as the sea quark effects are increased. It may indeed turn out that no meson in the physical spectrum is primarily a glueball - all states are mixtures of glue, $q\bar{q}$, $q\bar{q}q\bar{q}$, etc. We shall later discuss lattice results on the mixing of glueballs and scalar mesons (ie $q\bar{q}$ states).

In lattice studies, dimensionless ratios of quantities are obtained. To explore the glueball masses, it is appropriate to combine them with another very accurately measured quantity to have a dimensionless observable. Since the potential between static quarks is very accurately measured from the lattice, it is now conventional to use r_0 for this comparison. Here r_0 is implicitly defined by $r^2 dV(r)/dr = 1.65$ at $r = r_0$ where $V(r)$ is the potential energy between static quarks which is easy to determine accurately on the lattice. Conventionally $r_0 \approx 0.5$ fm.

Theoretical analysis indicates that for Wilson’s discretisation of the gauge fields in the quenched approximation, the dimensionless ratio mr_0 will differ from the continuum limit value by corrections of order a^2 . Thus in fig. 1 the mass of the $J^{PC}=0^{++}$ glueball is plotted versus the lattice spacing a^2 . The straight line then shows the continuum limit obtained by extrapolating to $a = 0$. As can be seen, there is essentially no need for data at even smaller a -values to further fix the continuum value. The value shown corresponds to $m(0^{++})r_0 = 4.33(5)$. Since several lattice groups ^{1,2,3,4} have measured these quantities, it is reassuring to see that the purely lattice observables are in excellent agreement. The publicised difference of quoted $m(0^{++})$ from UKQCD ³ and GF11 ⁴ comes entirely from relating quenched lattice measurements to values in GeV.

In the quenched approximation, different hadronic observables differ from experiment by factors of up to 10%. Thus using one quantity or another to

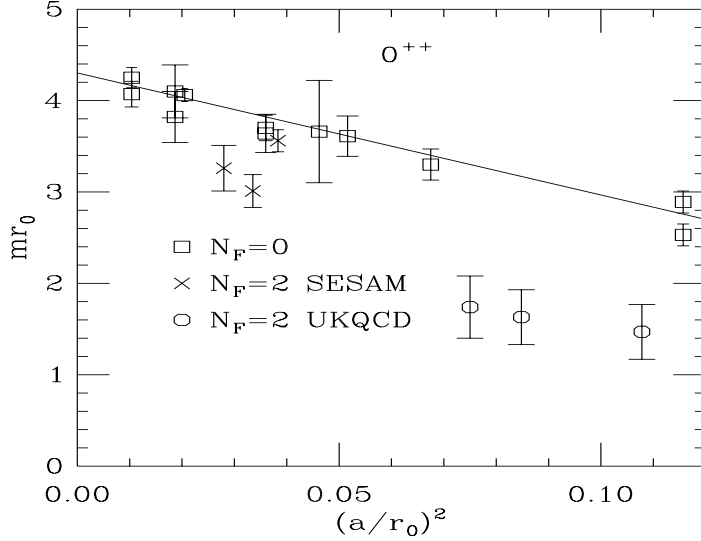


Figure 1: The value of mass of the $J^{PC} = 0^{++}$ glueball state from quenched data ($N_F = 0$)^{1,2,3,4} in units of r_0 where $r_0 \approx 0.5$ fm. The straight line shows a fit describing the approach to the continuum limit as $a \rightarrow 0$. Results^{5,20,6} with $N_F = 2$ flavours of sea quarks are also shown.

set the scale, gives an overall systematic error. Here I choose to set the scale by taking the conventional value of the string tension, $\sqrt{\sigma} = 0.44$ GeV, which then corresponds to $r_0^{-1} = 373$ MeV. An overall systematic error of 10% is then to be included to any extracted mass. This yields $m(0^{++}) = 1611(30)(160)$ MeV where the second error is the systematic scale error. Note that this is the glueball mass in the quenched approximation - in the real world significant mixing with $q\bar{q}$ states could modify this value substantially.

In the Wilson approach, the next lightest glueballs are^{2,3} the tensor $m(2^{++})r_0 = 6.0(6)$ (resulting in $m(2^{++}) = 2232(220)(220)$ MeV) and the pseudoscalar $m(2^{++})r_0 = 6.0(1.0)$. Although the Wilson discretisation provides a definitive study of the lightest (0^{++}) glueball in the continuum limit, other methods are competitive for the determination of the mass of heavier glueballs. Namely, using an improved gauge discretisation which has even smaller discretisation errors than the a^2 dependence of the Wilson discretisation, so allowing a relatively coarse lattice spacing a to be used. To extract mass values, one has to explore the time dependence of correlators and for this reason, it is optimum to use a relatively small time lattice spacing. Thus an asymmetric lattice spacing is most appropriate. The results⁷ are shown in fig. 2 and

for low lying states are that $m(0^{++})r_0 = 4.21(11)(4)$, $m(2^{++})r_0 = 5.85(2)(6)$, $m(0^{-+}) = 6.33(7)(6)$ and $m(1^{+-})r_0 = 7.18(4)(7)$. Another recent study ⁸ has used an improved discretisation based on the perfect action approach (without a space-time asymmetry) and obtains results consistent with earlier work.

One signal of great interest would be a glueball with J^{PC} not allowed for $q\bar{q}$ - a spin-exotic glueball or *oddball* - since it would not mix with $q\bar{q}$ states. These states are found ^{2,3,7} to be high lying: considerably above $2m(0^{++})$. Thus they are likely to be in a region very difficult to access unambiguously by experiment.

Within the quenched approximation, the glueball states are unmixed with $q\bar{q}$, $q\bar{q}q\bar{q}$, etc. Furthermore, the $q\bar{q}$ states have degenerate flavour singlet and non-singlet states in the quenched approximation. Once quark loops are allowed in the vacuum, for the flavour-singlet states of any given J^{PC} , there will be mixing between the $s\bar{s}$ state, the $u\bar{u} + d\bar{d}$ state and the glueball. One way to explore this is to measure directly the scalar mass eigenstates in a study with $N_f = 2$ flavours of sea-quark. Most studies show no significant change of the glueball spectrum as dynamical quark effects are added - but the sea quark masses used are still rather large ^{5,20}. A recent study ⁶, however, does find evidence for a reduced mass, albeit with a rather large lattice spacing, see fig. 1. This effect could be due to mixing of scalar mesons and glueballs, as we discuss below, or might just be a sign of an enhanced order a^2 correction at the relatively large lattice spacing used.

Let us now discuss the mixing of the scalar glueball and scalar mesons. The mass spectrum of $q\bar{q}$ states has been determined on a quenched lattice and the scalar mesons are found to lie somewhat lighter than the tensor states ⁹. These 2^{++} mesons are experimentally almost unmixed and so will be quite close to the quenched mass determination. This suggests that the quenched scalar masses from the lattice are at around 1.2 GeV and 1.5 GeV (for $n\bar{n}$ and $s\bar{s}$ respectively). An independent study ^{10,11} suggests that the scalar $s\bar{s}$ state is about 200 MeV lighter than the glueball which is a broadly compatible conclusion. Thus the glueball, at around 1.6 GeV, lies heavier than the lightest $q\bar{q}$ scalar states. This information can then be combined with mixing strengths to give the resulting scalar spectrum.

It is possible to measure the mixing strength on a quenched lattice even though no mixing actually occurs. On a rather coarse lattice ($a^{-1} \approx 1.2$ GeV), two groups have attempted this ^{11,6}. Their results expressed as the mixing for two degenerate quarks of mass around the strange quark mass are similar, namely $E \approx 0.3$ GeV ¹¹ and 0.5 GeV ⁶. From this evaluation of the mixing strength, one can use a mass matrix to estimate the mass shift induced in the glueball and scalar meson. The relevant mass matrix is (in a glueball, $q\bar{q}$ basis

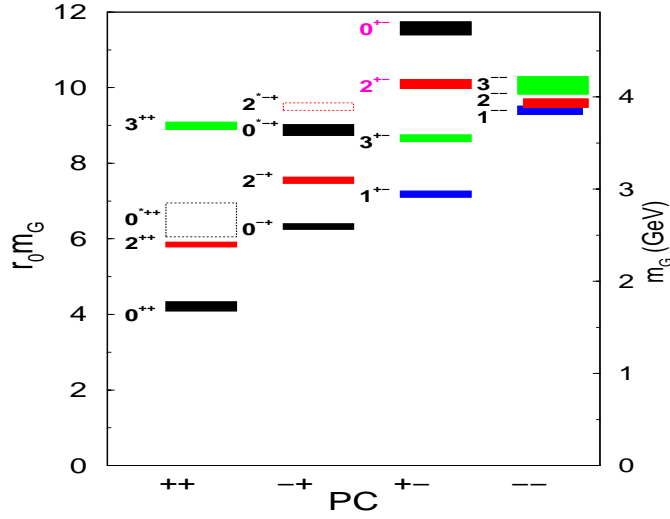


Figure 2: The continuum glueball spectrum⁷.

in GeV units):

$$\begin{pmatrix} 1.2 & 0.5 \\ 0.5 & 1.8 \end{pmatrix}$$

which would give a downward shift of the glueball mass by 20%. This is in qualitative agreement with our direct determination with $N_f = 2$ flavours of sea-quark that the lightest scalar mass is reduced significantly at this lattice spacing as shown in fig. 1.

Note that at this coarse lattice spacing the quenched glueball mass is reduced (see fig. 1) below the canonical value of 1.6 GeV. Thus a study at smaller lattice spacing is needed. An exploratory attempt to extrapolate to the continuum¹¹ gave a very small mixing of 86(64) MeV, while the other determination⁶ uses clover improvement so order a effects in the extrapolation to the continuum are suppressed and one would not expect a significant decrease in going to the continuum limit. What this discussion shows is that precision studies of the mixing on a lattice have not yet been achieved.

As well as this mixing of the glueball with $q\bar{q}$ states, there will be mixing with $q\bar{q}q\bar{q}$ states which will be responsible for the hadronic decays. A first attempt to study this¹² yields an estimated width for decay to two pseudoscalar mesons from the scalar glueball of order 100 MeV. A more realistic study would involve taking account of mixing with the $n\bar{n}$ and $s\bar{s}$ scalar mesons as well.

3 Exotic states

By exotic state we mean any state which is not dominantly a $q\bar{q}$ or qqq state. Such examples have been known for a long time: the deuteron is a proton-neutron molecule for example. It is very weakly bound (2 MeV) and is quite extended. Similar molecular states involving two mesons have been conjectured.

One case which is relatively easy to study is the BB system, idealised as two static quarks and two light quarks. Then a potential as a function of the separation R between the static quarks can be determined. Because the static quark spin is irrelevant, the states can be classified by the light quark spin and isospin. Lattice results ¹³ (using a light quark mass close to strange) have been obtained for the potential energy for $I_q = 0, 1$ and $S_q = 0, 1$. For very heavy quarks, a potential below $2M_B$ will imply binding of the BB molecules with these quantum numbers and $L = 0$. For the physically relevant case of b quarks of around 5 GeV, the kinetic energy will not be negligible and the binding energy of the BB molecular states is less clear cut. One way to estimate the kinetic energy for the BB case with reduced mass circa 2.5 GeV is to use analytic approximations to the potentials found. For example the $I_q, S_q = (0, 0)$ case shows a deep binding at $R = 0$ which can be approximated as a Coulomb potential of $-0.1/R$ in GeV units. This will give a di-meson binding energy of only 10 MeV. For the other interesting case, $(I_q, S_q) = (0, 1)$, a harmonic oscillator potential in the radial coordinate of form $-0.04[1 - (r-3)^2/4]$ in GeV units leads to a kinetic energy which completely cancels the potential energy minimum, leaving zero binding. This harmonic oscillator approximation lies above the estimate of the potential, so again we expect weak binding of the di-meson system.

Because of these very small values for the di-meson binding energies, one needs to retain corrections to the heavy quark approximation to make more definite predictions, since these corrections are known to be of magnitude 46 MeV from the B, B^* splitting. It will also be necessary to extrapolate the light quark mass from strange to the lighter u, d values to make more definite predictions about the binding of BB molecules.

Models for the binding of two B mesons involve, as in the case of the deuteron, pion exchange. The lattice study ¹³ is able to make a quantitative comparison of lattice pion exchange with the data described above and excellent agreement is obtained at larger R values, as expected.

4 Hybrid Mesons

By hybrid meson, I mean a meson in which the gluonic degrees of freedom are excited non-trivially. I first discuss hybrid mesons with static heavy quarks where the description can be thought of as an excited colour string. I then summarise the situation concerning light quark hybrid mesons.

Consider $Q\bar{Q}$ states with static quarks in which the gluonic contribution may be excited. We classify the gluonic fields according to the symmetries of the system. This discussion is very similar to the description of electron wave functions in diatomic molecules. The symmetries are (i) rotation around the separation axis z with representations labelled by J_z (ii) CP with representations labelled by $g(+)$ and $u(-)$ and (iii) CR . Here C interchanges Q and \bar{Q} , P is parity and R is a rotation of 180° about the mid-point around the y axis. The CR operation is only relevant to classify states with $J_z = 0$. The convention is to label states of $J_z = 0, 1, 2$ by Σ, Π, Δ respectively. The ground state ($\Sigma^+ - g$) will have $J_z = 0$ and $CP = +$.

The exploration of the energy levels of other representations has a long history in lattice studies ^{14,15}. The first excited state is found to be the Π_u . This can be visualised as the symmetry of a string bowed out in the x direction minus the same deflection in the $-x$ direction (plus another component of the two-dimensional representation with the transverse direction x replaced by y), corresponding to flux states from a lattice operator which is the difference of U-shaped paths from quark to antiquark of the form $\square - \sqcup$.

Recent lattice studies ¹⁶ have used an asymmetric space/time spacing which enables excited states to be determined comprehensively. These results confirm the finding that the Π_u excitation is the lowest lying and hence of most relevance to spectroscopy.

From the potential corresponding to these excited gluonic states, one can determine the spectrum of hybrid quarkonia using the Schrödinger equation in the Born-Oppenheimer approximation. This approximation will be good if the heavy quarks move very little in the time it takes for the potential between them to become established. More quantitatively, we require that the potential energy of gluonic excitation is much larger than the typical energy of orbital or radial excitation. This is indeed the case ¹⁴, especially for b quarks. Another nice feature of this approach is that the self energy of the static sources cancels in the energy difference between this hybrid state and the $Q\bar{Q}$ states. Thus the lattice approach gives directly the excitation energy of each gluonic excitation.

The Π_u symmetry state corresponds to excitations of the gluonic field in quarkonium called magnetic (with $L^{PC} = 1^{+-}$) and pseudo-electric (with 1^{-+}) in contrast to the usual P-wave orbital excitation which has $L^{PC} = 1^{--}$. Thus

we expect different quantum number assignments from those of the gluonic ground state. Indeed combining with the heavy quark spins, we get a degenerate set of 8 states with $J^{PC} = 1^{--}, 0^{-+}, 1^{-+}, 2^{-+}$ and $1^{++}, 0^{+-}, 1^{+-}, 2^{+-}$ respectively. Note that of these, $J^{PC} = 1^{-+}, 0^{+-}$ and 2^{+-} are spin-exotic and hence will not mix with $Q\bar{Q}$ states. They thus form a very attractive goal for experimental searches for hybrid mesons.

The eightfold degeneracy of the static approach will be broken by various corrections. As an example, one of the eight degenerate hybrid states is a pseudoscalar with the heavy quarks in a spin triplet. This has the same overall quantum numbers as the S-wave $Q\bar{Q}$ state (η_b) which, however, has the heavy quarks in a spin singlet. So any mixing between these states must be mediated by spin dependent interactions. These spin dependent interactions will be smaller for heavier quarks. It is of interest to establish the strength of these effects for b and c quarks. Another topic of interest is the splitting between the spin exotic hybrids which will come from the different energies of the magnetic and pseudo-electric gluonic excitations.

One way to go beyond the static approach is to use the NRQCD approximation which then enables the spin dependent effects to be explored. One study ¹⁶ finds that the $L^{PC} = 1^{+-}$ and 1^{-+} excitations have no statistically significant splitting although the 1^{+-} excitation does lie a little lighter. This would imply, after adding in heavy quark spin, that the $J^{PC} = 1^{-+}$ hybrid was the lightest spin exotic. Also a relatively large spin splitting was found ¹⁷ among the triplet states considering, however, only magnetic gluonic excitations.

Confirmation of the ordering of the spin exotic states also comes from lattice studies with propagating quarks ^{9,18,19} which are able to measure masses for all 8 states. We discuss this evidence in more detail below.

Within the quenched approximation, the lattice evidence for $b\bar{b}$ quarks points to a lightest hybrid spin exotic with $J^{PC} = 1^{-+}$ at an energy given by $(m_H - m_{2S})r_0 = 1.8$ (static potential ¹⁵); 1.9 (static potential ¹⁶, NRQCD ¹⁷); 2.0 (NRQCD ¹⁶). These results can be summarised as $(m_H - m_{2S})r_0 = 1.9 \pm 0.1$. Using the experimental mass of the $\Upsilon(2S)$, this implies that the lightest spin exotic hybrid is at $m_H = 10.73(7)$ GeV including a 10% scale error. Above this energy there will be many more hybrid states, many of which will be spin exotic. A discussion of hybrid decay channels has been given ²⁴.

The excited gluonic static potential has also been determined including sea quarks ($N_f = 2$ flavours) and no significant difference is seen ²⁰ - see fig. 3. Thus the quenched estimates given above are not superseded.

I now focus on lattice results for hybrid mesons made from light quarks using fully relativistic propagating quarks. There will be no mixing with $q\bar{q}$

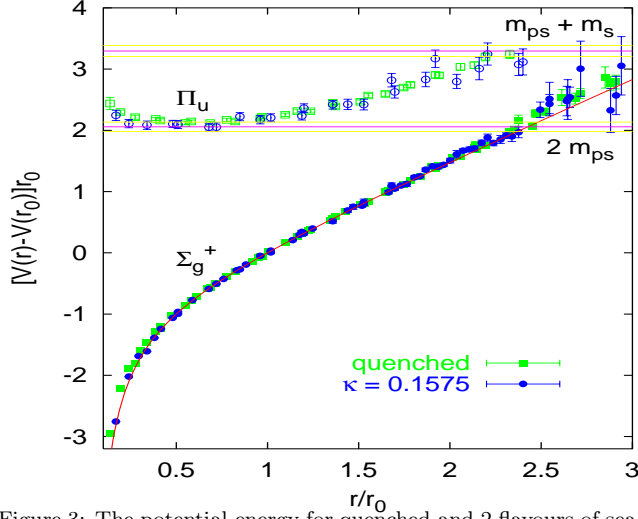


Figure 3: The potential energy for quenched and 2 flavours of sea quark for the ground state and first excited gluonic state ²⁰.

mesons for spin-exotic hybrid mesons and these are of special interest. The first study of this area was by the UKQCD Collaboration ⁹ who used operators motivated by the heavy quark studies referred to above to study all 8 J^{PC} values coming from $L^{PC} = 1^{+-}$ and 1^{-+} excitations. The resulting mass spectrum gives the $J^{PC} = 1^{-+}$ state as the lightest spin-exotic state. Taking account of the systematic scale errors in the lattice determination, a mass of 2000(200) MeV is quoted for this hybrid meson with $s\bar{s}$ light quarks. Although not directly measured, the corresponding light quark hybrid meson would be expected to be around 120 MeV lighter.

A second lattice group has also evaluated hybrid meson spectra with propagating quarks from quenched lattices. They obtain ¹⁸ masses of the 1^{-+} state with statistical and various systematic errors of 1970(90)(300) MeV, 2170(80)(100)(100) MeV and 4390(80)(200) MeV for $n\bar{n}$, $s\bar{s}$ and $c\bar{c}$ quarks respectively. For the 0^{+-} spin-exotic state they have a noisier signal but evidence that it is heavier. They also explore mixing matrix elements between spin-exotic hybrid states and 4 quark operators.

A first attempt has been made ¹⁹ to determine the hybrid meson spectrum using full QCD. The sea quarks used have several different masses and an extrapolation is made to the limit of physical sea quark masses, yielding a mass of 1.9(2) GeV for the lightest spin-exotic hybrid meson, which again is

found to be the 1^{-+} . In principle this calculation should take account of sea quark effects such as the mixing between such a hybrid meson and $q\bar{q}q\bar{q}$ states such as $\eta\pi$, although it is possible that the sea quark masses used are not light enough to explore these features.

The three independent lattice calculations of the light hybrid spectrum are in good agreement with each other. They imply that the natural energy range for spin-exotic hybrid mesons is around 1.9 GeV. The $J^{PC} = 1^{-+}$ state is found to be lightest. It is not easy to reconcile these lattice results with experimental indications²¹ for resonances at 1.4 GeV and 1.6 GeV, especially the lower mass value. Mixing with $q\bar{q}q\bar{q}$ states such as $\eta\pi$ is not included for realistic quark masses in the lattice calculations. This can be interpreted, dependent on one's viewpoint, as either that the lattice calculations are incomplete or as an indication that the experimental states may have an important meson-meson component in them.

5 String breaking

Although not directly related to exotic states as such, a discussion of string breaking is relevant to a study of the confinement mechanism. Here I concentrate on the situation of a static quark and antiquark at separation R with a gluonic field in a given representation labelled by J_z and CP where the separation axis is z . The ground state gluonic configuration has $J_z = 0$ and $CP = +$ (also $C\mathcal{R} = +$, see discussion above).

If a light quark and antiquark are to be produced, they must respect these symmetries. When these light quarks have no angular momentum about the separation axis, they will either be in a singlet state ($S_z = 0$, $CP = -$) or a triplet state ($S_z = 0$ or $S_z = 1$, $CP = +$). Thus the only possible transition is to the $S_z = 0$ triplet state. This is known as the 3P_0 model, although here it is exact. Having established the allowed transitions, I now discuss their strength.

Consider the energy of a static $Q\bar{Q}$ versus R for two different situations: (i) a gluonic flux in the ground state (this is the static potential $V(R)$) and (ii) the energy of two ground state mesons ($Q\bar{q}$ plus $\bar{Q}q$) at separation R (ie as B and \bar{B} mesons but with static heavy quark). We expect (i) to be lowest lying at small R and (ii) to be lowest lying at large R . Where these energies become degenerate is the string-breaking separation R_B . This is around 1.2 fm - see fig. 3. As has been emphasised in a study of adjoint string breaking²², the lattice enables string breaking to be studied as a mixing phenomenon.

In the quenched approximation there will be no string breaking but the matrix element can be estimated. In full QCD studies with sea quarks, string breaking is enabled and there will be mixing between these states. If this

mixing were substantial, the “potential energy” determined from Wilson loops should show a saturation at energy $V(R) = 2m_{Q\bar{q}}$ for $R > R_B$. Unfortunately, this effect has not yet been seen directly in lattice studies.

A comprehensive lattice study ²³ has been made with static quarks and light quarks of mass around the strange mass. This study finds that observation of the direct splitting at $R \approx R_B$ has errors too large to be conclusive. The magnitude of the mixing matrix element can, however, be evaluated with adequate accuracy. This suggests that indeed the mixing is weak (energy mixing strength of a few tens of MeV only). Such a weak mixing would explain why no direct evidence has been seen for string breaking from lattice studies of Wilson loops.

This discussion of string breaking can be extended ²⁴ to cover hybrid meson decays as well.

6 Conclusions

Quenched lattice QCD is well understood and accurate predictions in the continuum limit are increasingly becoming available. The lightest glueball is scalar with mass $m(0^{++}) = 1611(30)(160)$ MeV where the second error is an overall scale error. The excited glueball spectrum is known too. The quenched approximation also gives information on quark-antiquark scalar mesons and their mixing with glueballs. This determination of the mixing in the quenched approximation also sheds light on results for the spectrum directly in full QCD where the mixing will be enabled. There is also some lattice information on the hadronic decay amplitudes of glueballs.

Evidence exists for a possible $B_s B_s$ molecular state.

For hybrid mesons, there will be no mixing with $q\bar{q}$ for spin-exotic states and these are the most useful predictions. The $J^{PC} = 1^{-+}$ state is expected at 10.73(7) GeV for b quarks, 2.0(2) GeV for s quarks and 1.9(2) GeV for u , d quarks. Mixing of spin-exotic hybrids with $q\bar{q}q\bar{q}$ or equivalently with meson-meson is allowed and will modify the predictions from the quenched approximation.

String breaking at a separation of 1.2 fm has been studied and the breaking matrix element is found to be weak.

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